Evaluation of Measurement Uncertainty Based on the Propagation of Distributions Using Monte Carlo Integration.

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The Guide to the Expression of Uncertainty in Measurement [1], the so-called GUM, provides internationally agreed methods for the evaluation of measurement uncertainty. The GUM treats the subject in depth and provides theoretical justification. The latter is important, as the GUM introduces some concepts that are not necessarily new but have not been used widely before.

The GUM requests that for any measurement task a *model* be derived which mirrors the physical dependence of the measurand on all input quantities that may influence the measurand, and provides theoretically sound methods to treat uncertainties associated with the results of repeated measurements (Type A evaluation of uncertainty) and with any other measurement deviations (Type B evaluation). In either case one obtains a *probability distribution* that describes the knowledge about the quantity to be measured (the measurand). The expectation value of that distribution is taken as the best estimate of the measurand. The standard deviation of that distribution characterises the dispersion of the values that could reasonable be attributed to the measurand and is used as (the standard) uncertainty of the result of measurement.

In many or even most practical cases one does not have to consider these probability distributions explicitly, and it suffices to know their expectation values and standard deviations. The GUM provides an approach that, based on the aforementioned model, propagates the standard uncertainties associated with the input quantities. This approach is termed in the GUM the law of propagation of uncertainty.

However, there are measurement problems that lead to models for which the law of propagation of uncertainty is seen as insufficient.

This paper claims and shows that

- The propagation of the probability distributions associated with the input quantities provides a general approach for the evaluation of measurement uncertainty, and that
- Monte-Carlo integration is a nearly perfect tool for this task.

The paper features a tutorial part, which summarises the law of propagation of uncertainty and the Monte Carlo method, and a second part, which addresses some special features and, as we see it, the advantages of applying the Monte Carlo method.

In detail, discussion of the following topics is planned:

i. A brief discussion on deriving a model for the evaluation of measurement uncertainty
ii. A brief discussion on obtaining probability distributions associated with input quantities
iii. Propagation of distributions and the law of propagation of uncertainty
iv. Monte Carlo integration
v. Detailed treatment of an example by Monte Carlo integration and comparison with the law of propagation of uncertainty

In topic iv emphasis would be placed on the mathematical foundations as well as on practical aspects, *e.g.*, convergence of the Monte Carlo approach, and the consideration of mutually dependent input quantities.

The example used in topic v treats the frequently encountered problem of a spatially extended source and detector. It allows the advantages and disadvantages of the law of propagation of uncertainty and the Monte Carlo approach to be demonstrated.

[1] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, and OIML. Guide to the Expression of Uncertainty in Measurement, 1995. ISBN 92-67-10188-9, Second Edition.